Dealing with uncertainty in Earthquake Engineering: a discussion on the application of the Theory of Open Dynamical Systems

Enfrentando la incertidumbre en Ingeniería Sísmica: una discusión sobre la aplicación de la Teoría de Sistemas Dinámicos Abiertos

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Earthquakes, as a natural phenomenon and their consequences upon structures, have been addressed from deterministic, pseudo-empirical and primary statistical-probabilistic points of view. In the latter approach, 'primary' is meant to suggest that randomness has been artificially introduced into the variables of investigation. An alternative view has been advanced by a number of researchers that have classified earthquakes as chaotic from an ontological perspective. Their arguments are founded in the high degree of non-linearity of the equations ruling the corresponding seismic waves. However, the sensitivity of long time behavior of dynamic systems to variations in initial conditions, known as the Chaos Paradigm appears as a by-product of a deeper insight into natural phenomena known as Theory of Open Dynamical Systems ODS. An open system is currently defined as the relation between a part of nature, the main system which contains the observations we make, and its surrounding environment. ODS theory has been applied to different research subjects including physics, chemistry, and biology, for identifying and controlling undesired chaotic behavior in highly nonlinear dynamic systems. It is suggested that earthquakes and their interaction with structures constitute an example of an open system. Recognizing that in Earthquake Engineering the application of those concepts has not been previously investigated, in this paper a discussion related to the use of ODS concepts in

Los terremotos, como fenómeno natural y sus consecuencias en estructuras han sido abordados desde perspectivas deterministas, pseudo-empíricas y desde un punto de vista probabilístico-estadístico primario. En este último enfoque, el adjetivo 'primario' significa que la aleatoriedad ha sido introducida en forma artificial en las variables de investigación relevantes en Ingeniería Sísmica. Una perspectiva alternativa ha sido planteada por investigadores que han clasificado los terremotos, desde un punto de vista ontológico, como fenómenos caóticos. Sus argumentos se fundamentan en el alto grado de no-linealidad y complejidad de las ecuaciones que gobiernan la ecuación de las ondas sísmicas. Sin embargo, la sensibilidad de la respuesta dinámica en el largo plazo debido a pequeñas variaciones en las condiciones iniciales, conocido como el Paradigma del Caos, aparece como un subproducto de una mirada más profunda dentro de fenómenos naturales en general, llamada Teoría de Sistemas Dinámicos Abiertos ODS. Un sistema abierto se define como la relación entre una parte de la naturaleza, el sistema primario que contiene nuestras observaciones, y el sistema que lo contiene. Los principios de ODS han sido aplicados a distintas disciplinas científicas tales como la física, la química y la biología, con la inten-sión de identificar y controlar comportamientos caóticos no deseados en sistemas altamente no-lineales. Se postula que los terremotos y su interacción con estructuras constituyen un caso de un sistema abierto. Reconociendo que en Ingeniería Sísmica la aplicación de dichos conceptos no ha sido anteriormente investigada, en este artículo se presenta una discusión sobre su posible uso
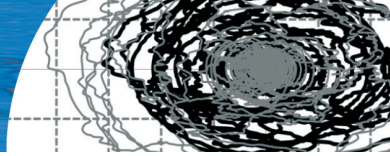
that particular field is presented. Using the most basic case of a linear elastic single degree of freedom SDOF oscillator, differences in the prediction of the response of the system subjected to only one ground motion using a Newton classical approach and ODS concepts, which involve stochastic processes, are compared. Conclusions about the consequences of the application of ODS theory for re-understanding Earthquake Engineering are presented, and a general critique to primary probabilistic approaches for addressing the same problem is formulated.

Keywords: structural dynamics, seismic uncertainty, chaos, open dynamical systems, seismic interaction

en dicha disciplina. Utilizando un modelo que representa el caso más básico de un oscilador de un grado de libertad con rigidez lineal elástica sometido sólo a un registro sísmico, se discuten las diferencias en la respuesta del oscilador obtenida usando la dinámica clásica de Newton y conceptos de ODS que incluyen procesos estocásticos como el que se utiliza en esta contribución. Conclusiones sobre las consecuencias de la aplicación de ODS son formuladas para re-entender la Ingeniería Sísmica, incorporando una crítica a enfoques probabilísticos primarios para abordar el mismo problema.

Palabras clave: dinámica de estructuras, incertidumbre sísmica, caos, sistemas dinámicos abiertos, interacción sísmica

Introduction

Earthquakes, in relation to Structural Engineering, have often been addressed from a primary statistical-probabilistic perspective. The use of 'primary' indicates that randomness has been artificially introduced within the variables of investigation. Alternative views have been advanced by a number of researchers that have classified earthquakes as chaotic from an ontological point of view (Strogatz, 1994). Their arguments are founded in the high degree of non-linearity and complexity of the equations ruling the corresponding wave dynamics.

Nevertheless, the sensitivity of large time behavior of dynamical systems to variations in initial conditions, the Chaos Paradigm (Lorenz, 1963), appears as a by-product of a deeper insight on natural phenomena, named the Theory of Open Dynamic Systems ODS (von Bertalanffy, 1950a,b). An open system is currently defined as the relation between a part of nature, the main system, inside which our observations are made, and its surrounding (unobserved) environment. Since the observed system and the environment are both in motion, as well as their exchanges of matter, energy, and information in general, there is a dynamic interaction. As such, an important consideration for the mathematical modeling of open systems is to determine the different time and space scales associated with the above collection of evolutions. In a classical context, the laws governing the dynamics of

mechanical systems are described by Newton's equations. Hence, once the initial conditions are fixed, the evolution of states (positions and moments) is fully determined. This is an example of a closed or isolated system. Unfortunately, it is impossible to completely isolate a part of nature; therefore a closed system cannot exist in reality. This is particularly true when studying seismic phenomena.

To the knowledge of the authors, no literature exists applying ODS to structural dynamics. In this paper, a preliminary philosophical discussion related to the use of these concepts in Earthquake Engineering in general and Structural Dynamics in particular, is presented. Focus is placed on discussing the uncertainty involved in predicting the response of structures subject to seismic events. An elastic and damped single degree of freedom SDOF system model is used, which represents a structure in the most simplistic way. It is shown that when applying stochastic perturbations, which represent the interaction of the open system (structure) with the environment (earth) in the form of a Lévy process, the dynamic response of that system under the same recorded ground motion exhibits significant variations when compared to the classical (unique) response, but it maintains the same order of magnitude. Important differences can also be appreciated when the proposed approach is compared to one which includes white noise only.

ODS fundamentals

In order to address the problem of the interaction of the open observed system with the containing environment, one may introduce a description of this relationship through variations in the initial conditions, as in chaos theory, or via perturbations of the equations of motion by stochastic processes. The selection of one or the other approach is a question of time scales. If one assumes that the main system evolves faster than the environment, then the prescription of their interaction via the initial conditions is plausible. On the other hand, if both system and environment interact almost simultaneously with their own evolution, then the second approach is more appropriate. Both methods are included in the development of ODS theory. The concept of an ODS has been in use for a long time. In physics, the concept appeared in the work of Boltzmann during the second half of the 19th century, remaining implicit in the development of statistical mechanics. However, in mathematics it only became an established theory in the 1970's. ODS include the so-called complex systems. Amongst mathematical approaches to them, the one based on Stochastic Processes (classical and quantum) is the most successful. As a result, the term Stochastic Analysis appears as a suitable envelope to describe a cross-disciplinary approach to ODS, (Fagnola *et al*., 1994; Gamble *et al*., 1999; Rey-Bellet, 2004; Rebolledo, 2004; Kossakowski and Rebolledo, 2007).

A critique of mechanistic uses of theory of probabilities in Earthquake Engineering

In recent years, some trends in Earthquake Engineering have explored the use of the theory of probabilities in the context of Performance-Based Earthquake Engineering PBEE as seen in the Vision 2000 document (SEAOC, 1995). In this context, probabilistic methods have been used for describing intensity measures IM, engineering demand parameters EDP, damage measures DM and decision variables DV in order to estimate a certain probability of exceedance in decision making for design.

These approaches have been developed in an increasingly sophisticated manner such that the level of physical background for stating the equations governing the models seems to have become a paradigm. Supporting physical background on the application of those concepts has been somehow forgotten, and philosophical reflections about the physical meaning of the complete process appear to have been jeopardized.

Bolotin (1960) divided the methods of analysis in Earthquake Engineering into three categories: (1) deterministic, (2) semi-empirical and (3) based on probabilistic methods. He proposed a fourth way inspired by the lack of information cumulated to that date, and the impossibility of a 'sharp' increase in the information in the near future. This fourth way consisted of assuming that the seismic action can be described by means of random functions of time which depend on a set of random parameters.

Most of the research done to date falls into the second and third categories (Anagnos and Kiremidjian, 1988; Mackie and Stojadinovic, 2004; Baker and Cornell, 2006; Uma *et al*., 2010). Research which deals with uncertainty can be found in Baker and Cornell (2008) for the estimation of losses due to earthquakes, and suggested the use of Monte Carlo simulations. In the work done by Suzuki and Minai (1988), SDE are implemented for describing the equation of motion of SDOF systems subjected to random-nature ground motions like that presented later by Ditlevsen and Bognár (1993) for elasto-plastic SDOF oscillators. The last approaches somehow fit the fourth category of Bolotin (1960). However, none of them has considered the interaction of the system under study with the containing environment, which is an open system dynamics problem.

State of the current practice

More recent trends in Earthquake Engineering have resulted in design codes which include probabilistic approaches in terms of factors for amplifying the amplitude of the design spectrum. This is the case of the 'return period factor' R in the New Zealand Standard NZS1107.5 (Standards New Zealand, 2004). According to the NZS1107.0:2002 (Standards New Zealand, 2002), $R = 1$ corresponds to an earthquake whose intensity has an annual probability of exceedence of 1/500 in 50 years of expected life, (ULS for Importance Level 2 buildings). The R factor then is increased for lesser probabilities of exceedance, imposing a larger spectral demand. For 'very rare' earthquakes a factor $R = 1.8$ is required and associated with a probability of exceedance of 1/2500 in 50 years.
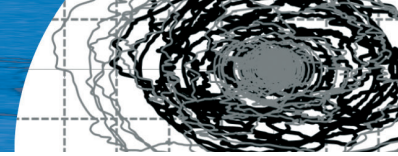
There are some philosophical problems with the latter approach. There is an ad-hoc assumption of randomness in the earthquake phenomenon which results in an ad-hoc use of randomness in engineering demand parameters. Additionally, deviations in the frequency content which characterizes the design spectrum of a site, defined by parameters like the corner period T_c of the displacement spectrum (Priestley *et al.*, 2007). It is worth noting that in Priestley *et al.* (2007), it is stated that the estimation of the parameter T_c is based on the statistics of different earthquakes which have occurred previously (semi-empirical).

The duration of the ground motion is not accounted for in modal spectral analysis, equivalent lateral force methods as well as nonlinear static methods (pushover), but it is accounted for in dynamic analysis methods (elastic or inelastic), where ground motion records are used. The relevance of the duration of the motion in a linear elastic dynamic analysis seems not to be as large as it can be in an inelastic analysis, the reason being that the mechanical properties of the structure vary during a nonlinear analysis, leading to completely different responses of a given structure when compared to the elastic counterpart model (Quintana-Gallo, 2008). Nevertheless, it is important to remember that a nonlinear numerical analysis can only be carried out after the design of the structure is ready. This method, in the context of displacement-based seismic design has been suggested for verification only. Therefore in current design only a snap-shot of the structure at a given limit state in terms of displacements (or forces) is being considered in the facts.

The estimation of the seismic intensity seems to have some philosophical issues. It is understood that the specification and the use of a certain way of representing a rational indicator of the seismic demand used for designing a structure is needed. It is recognized that the background for defining such an indicator is typically based on genuine information obtained during past earthquakes and based on considerations of the classical theory of dynamics of structures (Chopra, 2001). It is also recognized that efforts have been done in order to relate those indicators (numbers) to the damage observed during earthquakes where the data for developing that indicator has been obtained. However, no matter how much information can be gathered throughout the history in order to improve the accuracy of that indicator, there is always the possibility of that indicator to be refuted on the unsafe side. Assume that the indicator is correct: as the damage suffered by structures during two subsequent seismic events normally differ from one to the other, it follows that the a priori establishment of an expected damage is not possible. A posteriori, the damage can be evaluated; the demand parameters compared with the previous estimations, and start another trial and error process, which is nothing more than the conjectures and refutations method for science proposed by Karl Popper (1963). The important part is to acknowledge the fact that to err is to succeed.

Ground motion recordings from historical earthquakes and spectral response

Ground motions due to earthquakes have been recorded since a relatively 'short time' in the human civilization time-scale, which is a 'very short time' in geological time-scale. Ground motion records of strong earthquakes are reported in the literature since the 1940's (Housner, 1947). The spectral response of those records and some that may have occurred in between, are reported in Housner *et al.* (1953), even though the concept of response spectra was introduced before by Biot (1943). From there onward, the exponential increasing in the technology has allowed seismic records to be obtained in many more locations and with better precision.

Derivations for a 'mean' or 'reasonably accurate' characteristic spectrum for a given location have been done for a long time, sometimes in a highly sophisticated fashion. Relatively recent attempts for considering the factual uncertainty in the spectral response to be used at a certain site, have used the theory of probabilities for addressing the problem, based on statistical inference (Baker and Cornell, 2006). In the following sections, one arbitrarily selected ground motion recorded during the Loma Prieta earthquake in Gilroy Array#5 station is used. As will be discussed later, the choice of a certain record may be almost irrelevant.

Equation of motion of a closed sdof system under ground motion excitations

The equation of motion for a damped single degree of

freedom SDOF mass with elastic restoring (stiffness) and viscous damping (dissipating) forces, with applications for structural earthquake engineering is well known. The body of mass m representing the system's inertial mass is connected to the containing system (environment) by means of a spring of stiffness k and a viscous damper with equivalent damping ratio d. This is illustrated in Figure 1.
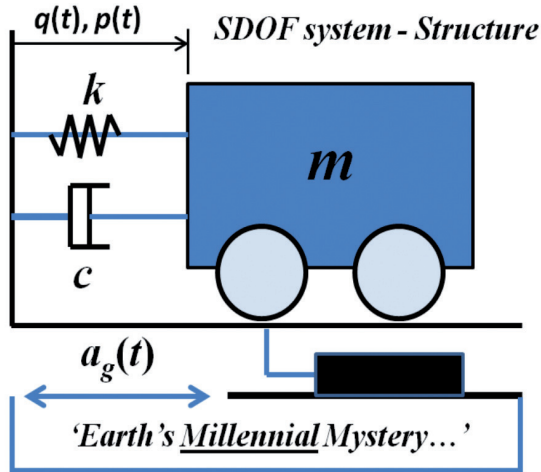

Figure 1: Single degree of freedom system with viscous damping subjected to earthquake ground accelerations

The equation which describes the motion of that system (response) under ground motion acceleration is given by equation (1), which can be re-written as equation (2).

$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = -ma_g(t) \tag{1}$$

$$\ddot{q}(t) + 2dv_n\,\dot{q}(t) + v_n^2 q(t) = -a_g(t) \tag{2}$$

where m is the inertial mass; c is the viscous damping; k is the stiffness; $a_g(t)$ is the ground acceleration; d is the critical damping; and v_n is the system natural frequency of vibration. The term $q(t)$ denotes the system response in terms of position and its derivatives with respect to time are denoted by dots (system velocity and relative acceleration using single and double dots, respectively). The above classical equations may be synthesized as a first order system by introducing the vector variable

$$x(t) = \begin{pmatrix} q(t) \\ p(t) \end{pmatrix},$$

where $p(t) = mdq(t)/dt$ denotes the momentum function,

and q_0, v_0, and p_0 are the initial conditions of position, velocity and momentum, respectively. Re-writing equation (1) in terms of $x(t)$, it becomes equation (3).

$$\dot{x}(t) = Mx(t) + b(t) \tag{3}$$

where,

$$M = \begin{bmatrix} 0 & 1 \\ -k & -\frac{c}{m} \end{bmatrix} \quad \text{and} \quad b(t) = \begin{Bmatrix} 0 \\ -ma_g(t) \end{Bmatrix}$$

Equation of motion of the sdof system from an open dynamical systems perspective

In Structural Dynamics the basic equation of motion of a SDOF system has been stated from Newton's mechanics. In this section, the problem is addressed from a different perspective which includes the interaction of the system with the surrounding environment. This interaction corresponds to an exchange of energy or momentum in this particular case.

The space of trajectories

Interactions with the environment may produce important variations on the whole trajectory of the system ruled by (3). In order to construct the appropriate mathematical model, we need to consider the set Ω of all possible solutions of (3) denoted by ω: $[0;\infty[\to |R^2$. Therefore, each element $\omega \in \Omega$ is a function of time, and for each $t \geq 0$, $\omega(t)$ is a vector in $|R^2$ such that:

$$\omega(t) = \begin{pmatrix} \omega_q(t) \\ \omega_p(t) \end{pmatrix}$$

Let us define the function X_t: $\Omega \to |R^2$, which corresponds to the state of the system, when following the trajectory ω. We also define Q_t: $\Omega \to R$, and P_t: $\Omega \to R$ as the functions of the position and momentum, respectively, so that $X_t = [Q_t, P_t]^T$. A trajectory $\omega \in \Omega$ corresponds to a possible solution of (3) if it satisfies (4) for all $t \geq 0$. Equation (4) corresponds to the integrated form of the solution, which can also be expressed in the differential form of equation (5).

$$X_t(\omega) = X_0(\omega) + \int_0^t (M\,X_s(\omega) + b(s))\,ds \tag{4}$$

$$dX_t(\omega) = \big(MX_t(\omega) + b(t)\big)dt \tag{5}$$
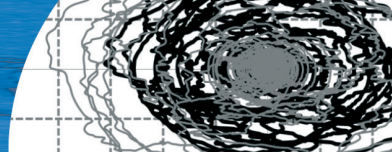
This formulation is still limited to a closed system. In the next section the mathematical formulation as well as the physical background for describing the interaction of an open system with the containing environment is explained.

Opening the System

Let us now choose our basic space of trajectories Ω to allow for discontinuities, which enables the possibility of the system to experience impulses or sudden variations in the momentum. Assume that at a given time $t = t_0$ our system (or particle) collides with a number of small objects or particles. These collisions introduce instantaneous modifications on the momentum (force). Mathematically that variation on the momentum is given by a jump in the function of the momentum at time t_0, such that: $\Delta P_{t0}(\omega) = P_{t0}(\omega) - P_{t0^-}(\omega)$. From a physical point of view, we have changed our system so that we no longer have a single particle (body) but a two-particle (body) system.

In the new system the jump in the momentum of the first body is (-1) times the jump in the momentum of the second body, following the principle of conservation in the momentum. Assume that the magnitude of the jump in the momentum of the colliding body is $\gamma > 0$. Name $\zeta(\omega)$ its sign, such that $\zeta(\omega) = 1$ if the main body is pushed forward, and $\zeta(\omega) = -1$ if it is pushed backwards in this simple one-dimensional example. The jump in the momentum at $t = t_0$ in terms of the function defined previously is:

$$\Delta P_{t0}(\omega) = \zeta(\omega)\gamma = \gamma \Delta V_{t0}(\omega)$$

where $\Delta V_{t0}(\omega)$ is the jump in the momentum occurring at a certain time t_0, normalized by the magnitude. The continuous form of the function $V_t(\omega)$ is given by $V_t(\omega) = \zeta(\omega)1_{[t0, \infty[}$, where $1_{[t0, \infty[}$ is the Heaviside function at t_0, which takes a value of 1 from $t = t_0$ to infinity. The function $t \to V_t(\omega)$ has finite variations on bounded intervals of the real line. Integration of a right-continuous function f, with respect to V is understood as:

$$F(t) = \int_0^t f(s)dV_s(\omega) = \sum_{0<s\leq t} f(s)\Delta V_s(\omega)$$

which allows one to use the short-hand writing $dF = f(t)\,dV(t)$. Therefore, the equation of motion of the SDOF

system can be written as equation (6), with the initial conditions given by equation (7):

$$dX_t(\omega) = (MX_t(\omega) + b(t))dt + \sigma dV_t(\omega) \qquad (6)$$

$$X_0(\omega) = x_0 = [q_0; p_0]^T \qquad (7)$$

where $\sigma = [0; c]^T$ is the magnitude of the state perturbation in terms of an $|R^2$ vector similar to $b(t)$, the ground motion excitation vector.

Let us now introduce the time sequence $(T_n)_{n\in|N}$ such that $T_n = nh$ and $h \in |R^+$ is the series time-step. Assume that the sequence of impulses takes place at times $T_1^h < T_2^h < \ldots < T_n^h < \ldots < T_N^h$. Then the process V_t becomes:

$$V_t^h(\omega) = \sum_{n=0}^{\infty} \xi_n(\omega)1_{[T_n^h(\omega),\, T_{n+1}^h(\omega)[}(t) = \sum_{n=0}^{[t/h]} \zeta_n(\omega)$$

where the sequence $(\zeta_n(\omega))_{n\in|N}$ has values in $\{-1, 1\}$ for all n. Assume that the masses of the colliding particles (or bodies) are all identical, as well as their momentum γ, which is the classical assumption for Brownian motion. The dissipated energy during the collisions E is proportional to the square of the momentum, such that in this case:

$$E \propto \gamma^2 \sum_{n=1}^{\infty} |\zeta_n|^2 = \gamma^2 [\frac{t}{h}]$$

In order to keep the dissipated energy finite as $h \to 0$, we need to select γ proportional to $\sqrt{h}$, that is $\gamma = c\sqrt{h}$, where c is a constant. We now re-write (6) as equation (8) for the functional $X_t^h(\omega)$ where the superscript h has been added for recalling the relevance of this parameter.

$$X_t^h(\omega) = X_0^h(\omega) + \int_0^t (MX_s^h(\omega) + b(s))ds + \binom{0}{c}\sqrt{h}\, V_t^h(\omega) \qquad (8)$$

Now we are faced with the following problem: on one hand the dissipative energy is $c^2h[t/h]$, which is kept finite and tends to c^2t in the limit when $h \to 0$, but on the other hand there are currently no tools to prove that $X_t^h(\omega)$ converges. In order to cope with this problem it is necessary to modify the mathematical framework of the study by introducing probabilities via stochastic processes.

Application of probabilistic concepts and stochastic processes – classical white noise

In order to solve the limit problem stated in the previous sections, consider a probability measure |P on the space Ω for which the sequence $(\zeta_n(\omega))_{n\in|N}$ satisfies the following conditions:

- ζ_n is |P-independent of ζ_m for all *m, n*;
- $|P(\zeta_n = \pm 1) = \frac{1}{2}$ for all *n*.

Under this hypotheses we obtain the characteristic function, or Fourier transform of $W_t^h = \sqrt{h} \cdot V_t^h$, given by:

$$|E\left(e^{iuW_t^h}\right) = \prod_{n=1}^{t/h} |E\left(e^{iu\sqrt{h}\zeta_n}\right) = \left(\cos(u\sqrt{h})\right)^{[t/h]}$$

In the limit when $h \to 0$, the last expression takes the form of

$$\lim_{h\to 0} E\left(e^{iuW_t^h}\right) = (1 - u^2h/2)^{[t/h]} = e^{-\frac{u^2t}{2}}$$

In Classical Probability Theory, the result stated above is known as the Central Limit Theorem for random variables of the form of W_t^h. The implication is that these variables converge in distribution towards a Gaussian (normal) random variable with zero mean and variance *t*. That result can be improved to prove that the family of processes W^h converges towards a Brownian motion (or Wiener noise), denoted as $(W_t, t \geq 0)$. It can also be proved that a limit equation of the form of equation (9) exists, which is a prototype of a stochastic differential equation (Rebolledo, 1979, 1980; Platen and Rebolledo, 1985).

$$dX_t(\omega) = (MX_t(\omega) + b(t))dt + \binom{0}{c}dW_t(\omega) \qquad (9)$$

In equation (9), *c* corresponds to the so called diffusion coefficient. This kind of equation is suitable for representing interactions due to small impulses occurring very often. During a seismic event there will certainly be interactions of this kind between the structure and the ground (earth) during non-destructive earthquakes.

Nevertheless, the greatest damage are the result of (to date) unpredictable earthquakes, which liberate a huge amount of energy. These events are geologically comparable to those unpredictable major events related to non-periodic large eruptions in volcanoes, such as the Ruapehu volcano in New Zealand (Gamble *et al.*, 1999), a phenomenon which was explained using the theory of ODS.

Inspired by those findings, we select a more suitable kind of stochastic perturbation for implementation in structural dynamics. This is a Lévy process, which contains a Brownian motion portion as well as a discontinuous 'jump' process, understood to be more representative of the incidence of a major earthquake during the strong motion part of it. We consider the addition of a Poisson process to equation (9), as described in the next section.

Proposed Lévy process

Assume that during the strong part of the ground motion additional interactions in terms of variations in the momentum as well as the position due to the interaction with the environment are both distributed according to a Poisson process with parameter λ. This is the series of occurrence times that are given by $(\tau_n)_{n\in|N}$ such that these times are independent and exponentially distributed with parameter λ, such that their mean is $1/\lambda$. In order to estimate this parameter one can take the mean value of the time interval between the times where these additional perturbations take place during the strong part of the ground motion.

Let us introduce the process N_t which counts the additional impulses: $N_t(\omega) = n$ if and only if $\tau_n(\omega) \leq t \leq \tau_{n+1}(\omega)$. We now define the magnitude of the interaction of *n*-th quake as $Z_n(\omega) = [Z_n^q(\omega); Z_n^p(\omega)]^T$, where $Z_n^q(\omega)$ and $Z_n^p(\omega)$ are the position and momentum components of the additional impulse magnitude vector $Z_n(\omega)$, respectively. If the sequences $(Z_n^q)_{n\in|N}$ and $(Z_n^p)_{n\in|N}$ are both independent (for the scope of this paper) and identically distributed (according to a certain probability distribution) and we further assume that these magnitudes are independent of N_t, then the process defined in equation (10) is a Lévy process.

$$L_t(\omega) = \begin{Bmatrix} L_t^q(\omega) \\ L_t^p(\omega) \end{Bmatrix} = \begin{Bmatrix} \displaystyle\sum_{n=0}^{N_t} Z_n^q(\omega) \\ \displaystyle\sum_{n=0}^{N_t} Z_n^p(\omega) \end{Bmatrix} \qquad (10)$$
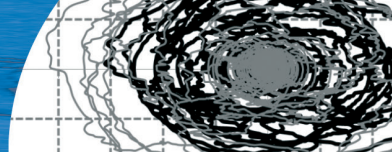
Adding the new set of interactions $L_t(\omega)$ into (9), we obtain equation (11), which represents a richer model for the open system dynamics, and it has a direct solution with the application of the Theorem 6.3.3 presented in Applebaum (2009).

$$dX_t(\omega) = (MX_t(\omega) + b(t))dt + \begin{pmatrix} 0 \\ c \end{pmatrix} dW_t(\omega) + dL_t(\omega) \tag{11}$$

A suitable numerical method for solving (11) is the Euler-Maruyana method (Kloeden and Platen, 1992), which can be simplified to solve (3).

Numerical simulation of the sdof system response using both approaches

In the following section, numerical simulations of the SDOF system response under study within the context of a classical and an open system dynamics approaches are presented. Only one seismic record was used, in light of the extremely unlikely event that the same ground motion occurs again in a future earthquake, and consequently the corresponding spectral response (Housner *et al.*, 1953; Goodman *et al.*, 1954; Bolotin, 1960).

Numerical solution of the equations of motion

For a given discretization of time $t_0 = T_0 < T_1 < \ldots < T_n = T_N$ of the time interval $[t_0; T_N]$ and time interval $\Delta t = h$, an Euler-Maruyana approximation of equation (11) is a continuous time stochastic process $Y = \{Y(t), t_0 \leq t \leq T_N\}$ which satisfies equation (12), for $n = 0$ to $N - 1$.

$$Y_{n+1} = Y_n + (M\,Y_n + b_n)h + \begin{pmatrix} 0 \\ c \end{pmatrix}(W_{T_{n+1}} - W_{T_n}) + (L_{T_{n+1}} - L_{T_n}) \tag{12}$$

Assume the natural period of vibration of the oscillator is 1 s, the inertial mass $m = 1$, and a critical damping $d = 5\%$. Under these conditions, $k = 4\pi^2$, and $c = \pi/5$, which determines the matrix M. Initial conditions of the process are set to zero such that $Y_0(\omega) = X_0 = x\,(t = 0) = x_0$.

In order to define the vector $b(t)$, the ground motion shown in Figure 2 was used, recorded at Gilroy Array #5 station, during the Loma Prieta earthquake (CA 1989). The amplitude of the original record was scaled to a PGA = 1g, and the shape 'adjusted' to fit the New Zealand Standard (Quintana-Gallo *et al.*, 2011). The earthquake ground

motion is determined for all $t_n = 0$, h, $2h\ldots \mathcal{F}$, where $\mathcal{F}$ corresponds to its duration, such that $\mathcal{F} \leq T_N$, allowing for a free vibration at the end of the response.
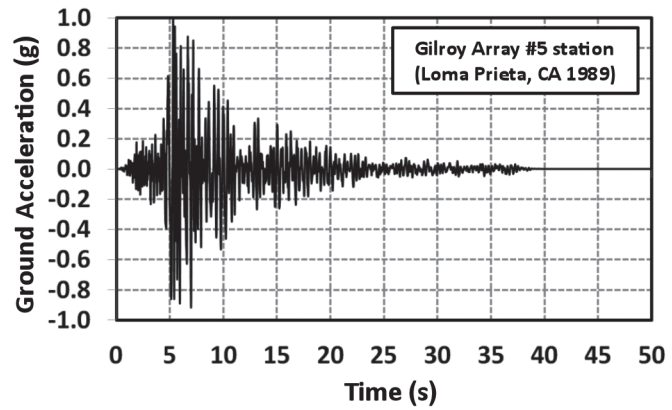

Figure 2: Ground motion acceleration input recorded at Gilroy array #5 station during Loma Prieta Earthquake, CA 1989

The response of the SDOF system in the context of classical dynamics is shown in Figure 3. The response is given in terms of displacements since they have been recognized to be one of the most important parameters in seismic design (Priestley, 1998, 2003; Priestley *et al.*, 2007) and also in terms of the phase state vector in time, described in the position – momentum domain. This response is unique from a mathematical perspective in the context of Newton's classical mechanics.

Open system responses with Brownian motion interaction (Wiener process)

The solution of the equation of motion of the system given in equation (12) includes a Brownian motion stochastic process only. This process also known as the Wiener process, was truncated at a time $\mathcal{F}_1 = 35$ s, when the ground motions approaches to rather small accelerations. From the infinity of responses that can be generated, two were selected and are presented in Figure 4. The coefficient of diffusion for Brownian motion is taken as $c = 1$.

In the graphs of Figure 4 it can be observed that, (1) the responses are all different from the classical counterpart, (2) they are all different from each other, (3) the shape remains similar and under certain limits, (4) the peak response is reached at approximately the same time in all cases, with fairly similar values. As will be show next, this is not the case when adding a complete Lévy process.
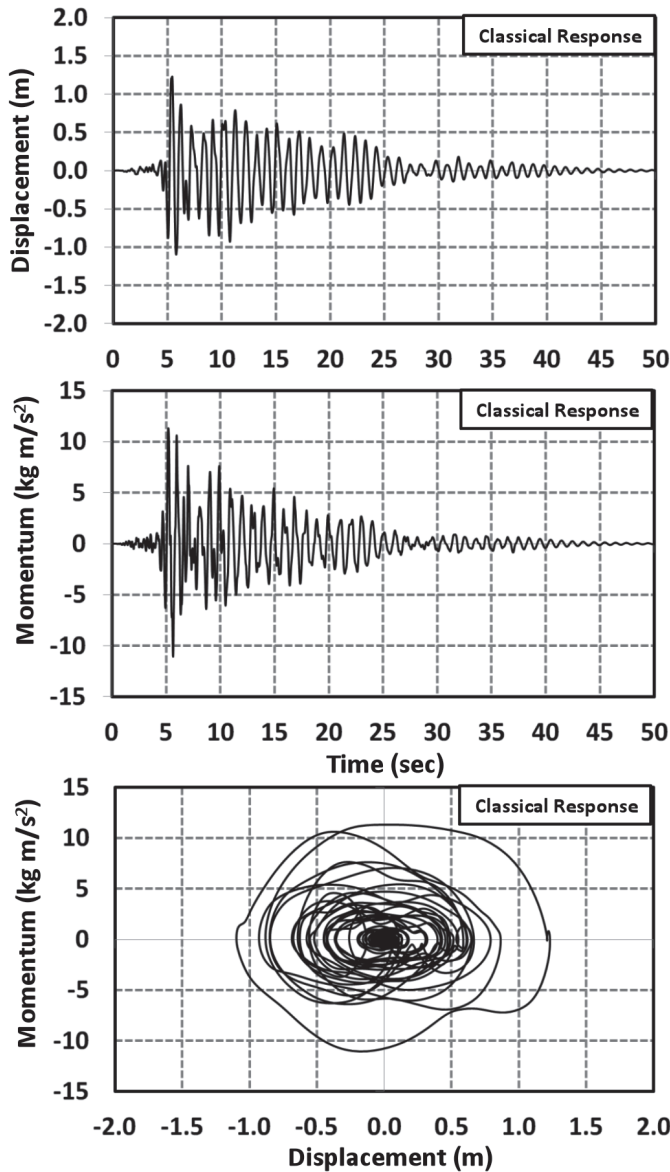
Figure 3: Numerical simulation of the response of the SDOF in a classical structural dynamics context

Figure 4: System response with additional Brownian motion only

Open system response with the proposed Lévy process interaction

In this section, the equation of motion with the complete Lévy process (Applebaum, 2004, 2009) was implemented. As this interaction is associated to that occurring during intense parts of the ground motion, when the exchange of energy is greater, they were associated to the strong motion part of the input motion only. This is in this case from 4.5 to 26 s as can be seen in Figure 2. Note that intensity in this case necessarily requires the existence of the system withs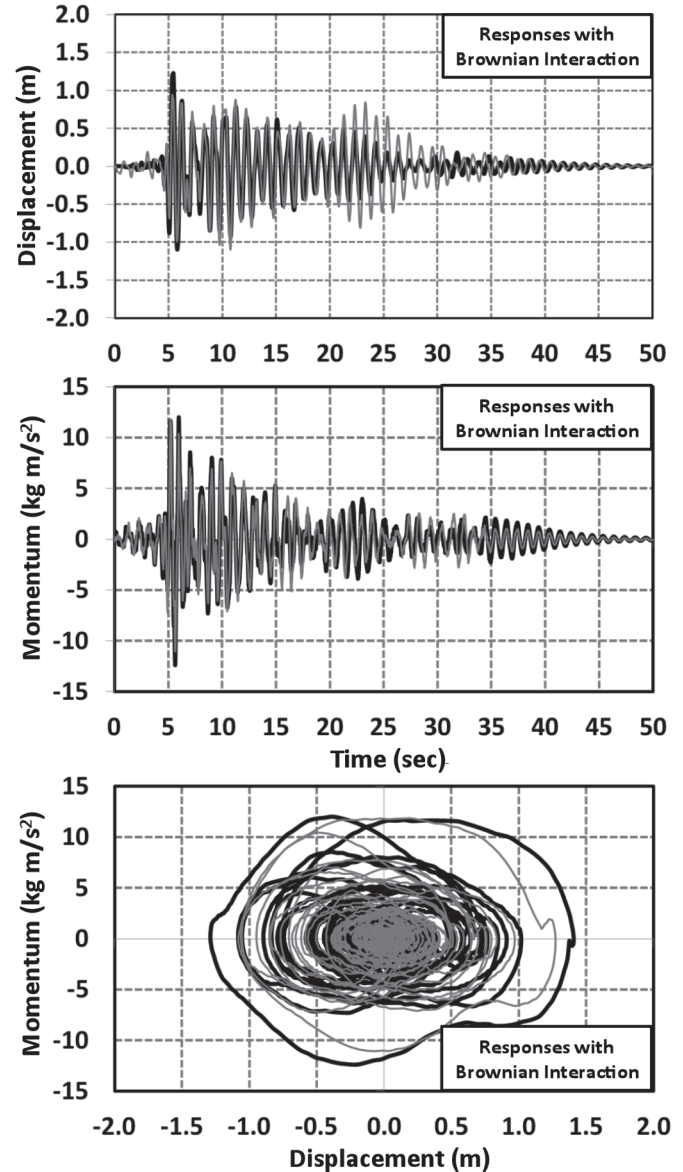tanding the actions imposed by seismic waves in dynamical interaction with the system, as was remarked at the beginning of this article.

For the scope of this paper, which deals with a primitive formulation of ODS concepts applied to Structural Dynamics, the main parameters have been set to arbitrary numbers, but keeping in mind the order of magnitude of those assumptions. In future contributions, parametric analysis of these parameters in order to evaluate their influence on the response of the open system, and empirical data is used for estimating the parameters in accordance with physics considerations.
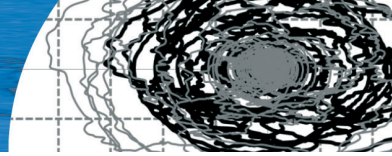
In this first approximation, the Poisson parameter was set to λ = 3, the magnitude of the displacement part of the magnitude was assumed to be represented by a normal distribution with mean value $Z_n^q = 0.01$ m and a standard deviation of 0.05 m (with the sign calculated as done for the Brownian motion), and the momentum part Z_n^p set to a Brownian component with coefficient $c = 1$ (see previous section).
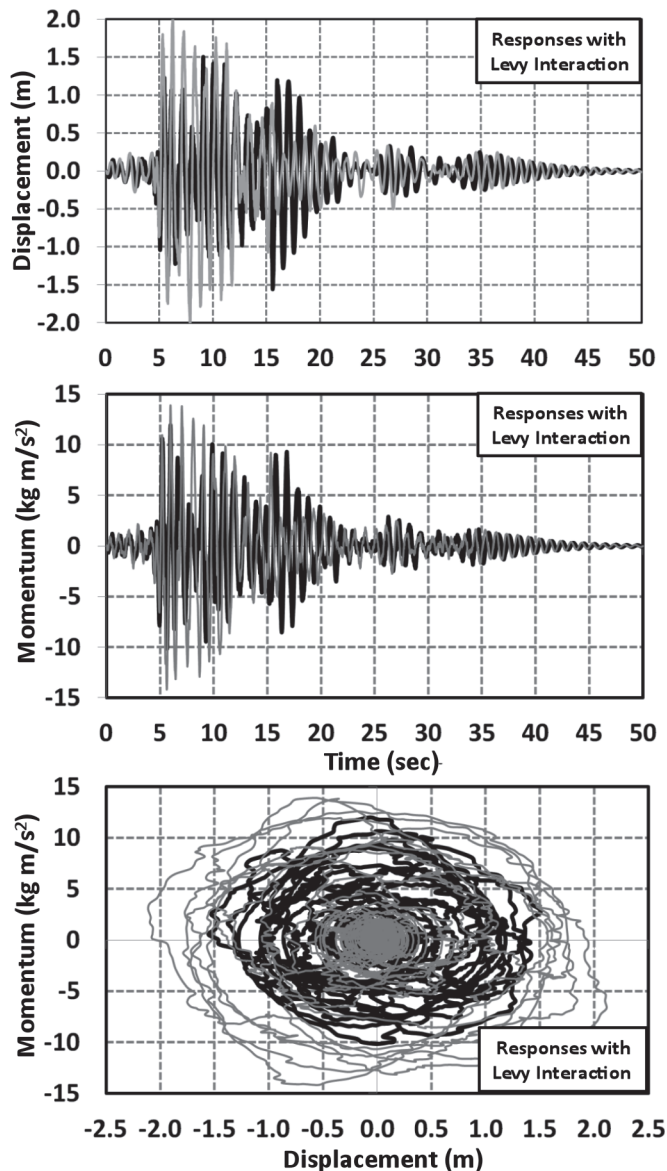

Figure 5: System response when using a Lévy process

In Figure 5, two of the infinite responses which represent a solution of equation (10) for the selected ground motion are presented. It can be observed that in this case: (1) the responses can differ significantly in shape from that predicted with only the classical and Brownian motion approaches, (2) the predicted amplitude of the response can be larger and smaller than those predicted using the other two approaches, (3) two system's responses under the same process can differ significantly in shape and magnitude between each other.

The two responses presented in Figure 5 seem to be associated in nature to a large extent to the unpredictable, but delimited inside some limits at the same time. The latter can be understood in common terms as chaotic behavior. In this case, the chaotic behavior is a consequence of the interaction of the observed system with the containing environment, which must be accounted for in ODS.

Re-understanding Earthquake Engineering from an ODS perspective

We can understand Earthquake Engineering as the discipline which deals with the design of structures to withstand earthquakes of an uncertain nature safely. This requires the understanding of (at a minimum) Structural Dynamics and Seismology. Seismology is the discipline of Geological Sciences which deals with tectonic processes and earthquakes, and is expected to provide information about the seismic hazard at a given location.

Hazard analysis has been largely studied in the past and is still a matter of great discussion. Most of the time it is implicitly assumed that the most suitable records for a certain region correspond to those recorded previously on the same site. The intensity of these records is related normally to PGA or maximum S_a indicators, which are based on mean values recorded in the past. The latter can be described as a mix of Bolotin's (1960) second and third categories: semi-empirical and probabilistic, if conditional probabilities are being used.

Nevertheless, new earthquake events like those that occurred in Chile (27 February 2010) and New Zealand (22 February 2011) keep refuting our expectations based on what was recorded and observed before in the same place in terms of recorded ground motions and consequences (intensity) (Quintana-Gallo *et al.*, 2013a,b). We re-understand Earthquake Engineering by reviewing the Structural Dynamics behind a simple SDOF oscillator from an ODS perspective. We argue that there is not a

'unique' fully determined structural response of a SDOF subjected to a unique record, but there are many, like in quantum mechanics.

Conclusions

Inspired by the uncertainty involved in the structural engineering design process, the theory of open systems has been applied to Structural Engineering in a very basic way. The findings described in the previous sections open a new area of investigation in order to identify and mitigate the sources of uncertainty in Structural Dynamics. We believe that future research in this direction is plausible, in light of the arguments presented.

We argue that one may not be taking into account a large amount of complexity and uncertainty inherent to the problem of structural dynamics in current design, because we deal with closed systems which are not necessarily representative of reality. Therefore the revision of some earthquake engineering concepts by recalling its fundamentals and recognizing the uncertainty involved in the process needs to be addressed from alternative approaches. It is concluded that ODS represents an important tool for future investigations in the field of Earthquake Engineering, and can provide great insight into the uncertainty involved in the discipline.

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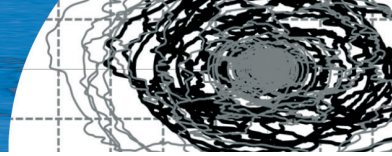
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