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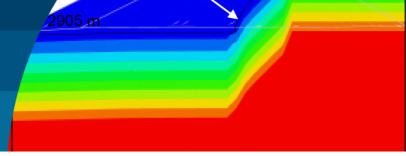

Comparative study of P2PSand and Mohr-Coulomb constitutive models using FLAC3D for the seismic modeling of a tailings dam in Chile

Estudio comparativo de los modelos constitutivos P2PSand y Mohr-Coulomb utilizando FLAC3D para el modelamiento sísmico de un depósito de relaves en Chile

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The trend in the dynamic analysis of tailings deposits is to use advanced constitutive models in order to properly capture the cyclic behavior of the materials and thus have a better estimation of the physical stability of these structures. In this context, the main objective of this research is to compare the results of numerical simulations performed with a constitutive model used in current practice, such as the Mohr-Coulomb elasto-plastic model, and an advanced constitutive model based on the critical state theory. This model, P2PSand, has the ability to represent the contractive and dilatant behavior of soils and the essential characteristics of the cyclic behavior of sands. From the results of the numerical analysis, the following conclusions can be drawn: P2PSand is able to adequately simulate the dynamic behavior of the sand in the tailing dam subjected to a measured earthquake of medium intensity (M_w = 5.4). P2PSand and Mohr-Coulomb constitutive models have the capability to analyze a maximum credible earthquake of magnitude 8.0 (M_w = 8.0), even if the earthquake is increased by 1.5 and 2 and if an advanced constitutive model is not used, the use of an elastic-plastic model with the addition of hysteretic damping is recommended.

Keywords: tailings dam, liquefaction, P2PSand, Mohr-Coulomb, FLAC3D, seismic

La tendencia en el análisis dinámico de depósitos de relaves es utilizar modelos constitutivos avanzados con el fin de capturar adecuadamente el comportamiento cíclico de los materiales y así tener una mejor estimación de la estabilidad física de estas estructuras. En este contexto, el objetivo principal de esta investigación es comparar los resultados de simulaciones numéricas realizadas con un modelo constitutivo utilizado en la práctica actual, como es el modelo elasto-plástico de Mohr-Coulomb, y un modelo constitutivo avanzado basado en la teoría del estado crítico. Este modelo, P2PSand, tiene la capacidad de representar el comportamiento contractivo y dilatante de los suelos y las características esenciales del comportamiento cíclico de las arenas. A partir de los resultados del análisis numérico, se pueden extraer las siguientes conclusiones: P2PSand es capaz de simular adecuadamente el comportamiento dinámico de la arena en la presa de relaves sometida a un sismo medido de intensidad media (M_w = 5.4). Los modelos constitutivos P2PSand y Mohr-Coulomb permiten analizar un sismo máximo creíble de magnitud 8.0 (M_w = 8.0) e incluso, si el sismo se incrementa en 1.5 y 2 y si no se utiliza un modelo constitutivo avanzado, se recomienda usar un modelo elasto-plástico con amortiguamiento histerético.

Palabras clave: presa de relaves, licuefacción, P2PSand, Mohr-Coulomb, FLAC3D, sísmica

Introduction

The Chilean government enforces the physical stability of tailings storage facilities (TSF) under static and dynamic conditions through the SERNAGEOMIN (National Geology and Mining Service). Considering that Chile has a high seismicity and some of the largest tailings dams, numerical analyses are essential to guarantee their physical stability. In this context, the current trend for numerical modeling of tailings dams seeks to improve the dynamic analyses by using advanced constitutive models that properly represent the materials' cyclic behavior. Nowadays, several constitutive models describe the performance of granular materials, including liquefaction as a relevant factor to study in dynamic analyses. Although both UBCSand (Beaty and Byrne, 2011) and SANISand (Dafalias and Manzari, 2004) models present outstanding merits, they still exhibit some limitations, including the damping overestimation at large deformations (Cheng, 2018). However, it is the P2PSand (Practical Two-Surface Plastic Sand Model) model (Cheng, 2018; Cheng and Detournay, 2021), a model specifically designed for geotechnical earthquake engineering applications, the one that partially addresses these limitations. This model modifies the SANISand one (Dafalias and Manzari, 2004) based on the critical state theory of soils. In this regard, one of the P2PSand features is a set of parameters used to simulate responses of different relative densities and initial stress states (Cheng, 2018) and to modify all void ratio-related formulas. P2PSand captures volumetric changes, pore pressure generation, and energy dissipation appropriately. The Mohr-Coulomb (M-C) elastoplastic model can simulate tailing dams in static conditions. In contrast, the Finn/Byrne constitutive model (Byrne, 1991) can simulate materials susceptible to liquefaction in dynamic conditions to better model (in zones below groundwater level) the pore pressure generation due to the volumetric deformation of materials. This study aims to compare the results of numerical simulations for the dynamic performance of a tailing dam, considering the Mohr-Coulomb elastoplastic model and the advanced constitutive model P2PSand (Cheng and Detournay, 2021) implemented in the numerical analysis software FLAC3D (2019).

Three-dimensional numerical model construction

The finite difference software FLAC3D (2019), version 7.0, was used for this model. Figure 1 shows the FLAC3D model's isometric view and visualization section. Stability models developed for this work required incorporating hydrogeological inputs for the correct performance of dynamic analyses. The phreatic surface (PS) analyzed across the wall for 2947.5 and 2955 masl elevations is shown in Figure 2. The PS at 2947 masl is the actual 2018 condition, while the PS at 2955 masl is considered a Maximum Probable Flood (MPF) condition and represents the maximum level the PS could reach in the reservoir basin as a consequence of the most significant precipitation event for the study area.

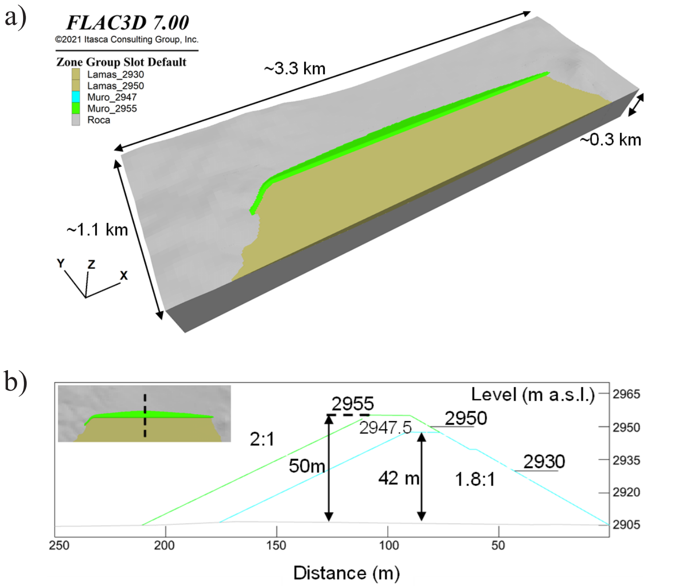

Figure 1: a) Isometric view of the FLAC3D model and b) visualization section

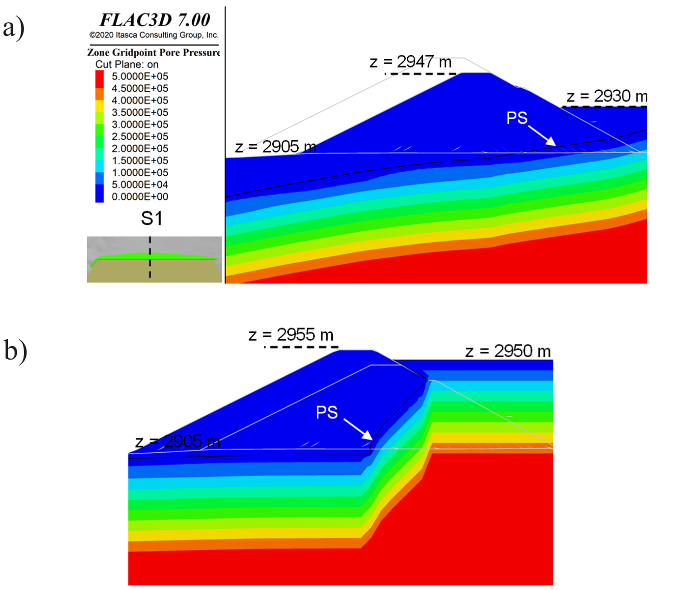

Figure 2: Pore pressure contours (limited to 0.5 MPa), 2947.5 masl and b) 2955 masl

Surface earthquakes recovery and material properties

The analyses were performed based on two seismic records. The first one corresponds to the largest earthquake recorded in 2018 at the TSF site area with a magnitude M_w = 5.4, and the second one is the Maximum Credible

Earthquake (MCE) proposed by a seismic hazard study. Figure 3 compares the original recorded (red) and the rock surface response (cyan) of the acceleration and pseudo-acceleration spectra. The figure shows an almost perfect agreement, implying that the model can reproduce the seismic records at the rock surface. In the same figure, the Fast Fourier Transform (FFT) and the foundation soil and transfer function (TF) of the dam, obtained as the ratio between the FFT at the dam crest and the FFT at the free field, are presented. The dam predominant frequency is approximately 3 Hz and derived from the Fourier spectra ratio smoothed by the Konno and Ohmachi (1998) method, which allows the predominant frequencies to be established. This frequency is similar to the frequency obtained from in-situ measurements using the HVR methodology.

The properties of the dam materials are summarized in Table 1. The numerical model explicitly includes the bedrock presence. The bedrock and tailings properties are shown in Table 2.

For sand material, hysteretic damping was added, as a complement to the Mohr-Coulomb constitutive model, in order to account for the degradation of the shear modulus and the increase of the damping level. The calibration of the hysteretic damping in FLAC3D is carried out using the function implemented by default.

Table 1: Dam properties

Parameter	Unit	Value
Dry density, water content	t/m³, %	1.94, 12
Shear strength: c, ϕ	kPa, °	0, 37
Maximum void ratio, minimum void ratio	-	0.75, 0.40
Relative density	%	79
Young modulus	kg/cm²	$326 + 642\log(\sigma'_3)$

Table 2: Bedrock and tailings properties

Material	Density, t/m³	Young modulus, kg/cm²	Poisson's ratio (-)	Undrained shear strength (-)
Bedrock	2.5	2×10^5	0.25	-
Tailings	1.8	$20\,(\sigma'_3)^{0.5}$	0.49	$\frac{s_u}{\sigma'_v} = 0.07$

P2PSand constitutive model

This model supports the critical state theory (it requires the critical state friction angle, φ_{cs}, as an input parameter) and

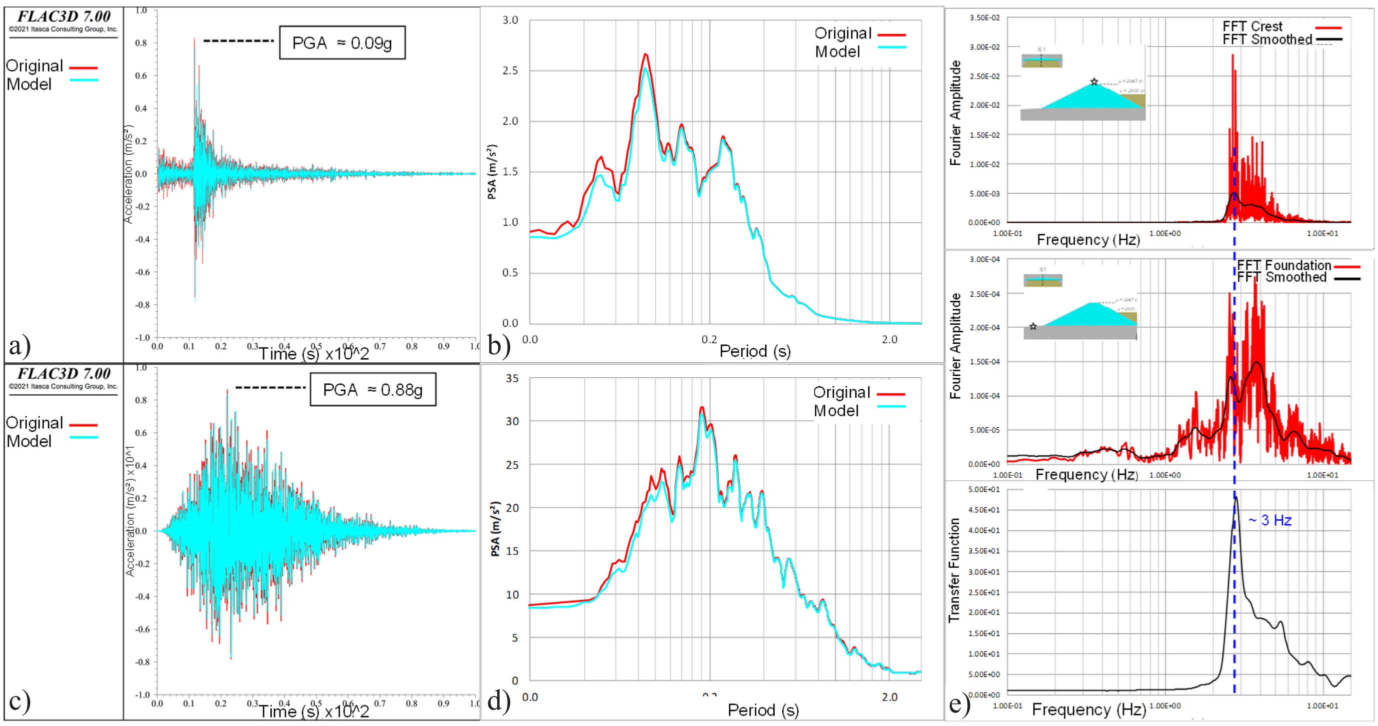

Figure 3: Earthquake $M_w = 5.3$: a) Horizontal acceleration and b) horizontal response spectrum. MCE $M_w = 8.3$: c) Horizontal acceleration and d) horizontal response spectrum. (e) Dam transfer function for earthquake $M_w = 5.3$.

uses the state parameter (distance between the initial Relative Density and the critical state line) to define whether the performance will be contractive or dilative. Therefore, P2PSand is based on the material's Relative Density (*D*r) and requires the minimum and maximum void ratios as input parameters. The critical state line is given as follows (two-parameters):

$$Dr_c = \frac{R}{Q - \ln\left(\frac{100 p_c}{p_{atm}}\right)} \quad (1)$$

where *Q* and *R* are Bolton's constants (1986) (material dependent and can be calibrated) with 10 and 1 default values, respectively. Alternatively, the critical state line can be defined using three-parameters as follows:

$$Dr_c = Dr_{c0} + \lambda_c \left(\frac{p_c}{p_{atm}}\right)^{\xi} \quad (2)$$

where Dr_c, λ_c and ξ are material parameters and ξ usually equal to 0.7 for sands. The elastic parameters are obtained as follows:

$$G = G_{ref} p_{atm} \left(\frac{p}{p_{atm}}\right)^{0.5}, \; K = \frac{2(1+\nu)}{3(1-2\nu)} G \quad (3)$$

where ν is the Poisson's ratio and G_{ref} is a density-dependent material parameter. G_{ref} is assumed to be a function of the relative density, Cheng (2018) has shown that linear function as follows:

$$G_{ref} = f(D_r) = g_0(D_r + C_{Dr}) \quad (4)$$

where g_0 and C_{Dr} are material constants with defaults values of $g_0 = 1240$ and $C_{Dr} = 0.01$.

Calibration of the constitutive model

This research compared simulations of the dam performance, considering the P2PSand and Mohr-Coulomb (M-C) constitutive models. To calibrate the elastic law of P2PSand for the sand, and comply with both the laboratory data, and for the purpose of a practice-oriented calibration with a set of parameters unique to the sand under study, the elastic parameters calibrated in this work were $g_0 = 800$ and $C_{Dr} = 0$. With these parameters, a good fit of the elastic branch of the stress-strain curve was obtained. For the critical state calibration, a numerical calibration exercise was carried out on triaxial tests (100, 300 and 900 kPa) selected from those developed by a soil mechanics laboratory for tailings sands, aimed at reproducing laboratory results.

Figure 4 shows the calibration of the critical state considering the best fit ($Q = 9.8$ and $R = 1.6$) with respect to the laboratory tests. According to the sensitivities carried out, it was observed that the *Q* parameter controls the resistance peak, while the *R* parameter is associated with the volumetric behavior. In the same figure shows how the stress-strain curves of those models compare with the results obtained from the laboratory tests for 300 kPa confinement (selected). In the absence of an advanced constitutive model, an elastoplastic model is recommended. Note that the M-C model successfully replicates the performance of the laboratory triaxial tests for large deformations. Moreover, the simulation shows the absence of a dilatancy characteristic for the M-C model, and the K_{cyc} factor (factor-cyclic) is a parameter to calibrate the liquefaction potential of the P2PSand, which controls the cyclic material performance and allows for a cyclic mobility or liquefaction adjustment (Cheng, 2018). Data from single cyclic shear (Cyc-Dss) tests (undrained condition) at two confining pressures (50 and 300 kPa) for 0.1% and 1% strain levels were considered a reliable calibration. It looks to adjust a single cyclic factor to ensure a similar performance of the numerical specimens compared to the laboratory samples for each confinement and deformation level. Figure 4 shows the results obtained for the 300 kPa confinement and controlled deformation level of 1% when compared, showing the performance of the FLAC3D model (in cyan) and the laboratory test (blue for the first cycle and red for the last cycle) under the numerical calibration scheme at controlled deformation.

The calibration exercise to obtain the critical state values dependent on three parameters was to plot a curve of average stress as a function of the *Dr* for two-parameters (*Q* and *R*) already calibrated in the previous stage ($Q = 9.8$ and $R = 1.6$), and then through a process iteratively fit the same curve in the best way, based on the three-parameter dependent critical state equation, as shown in Figure 5. An

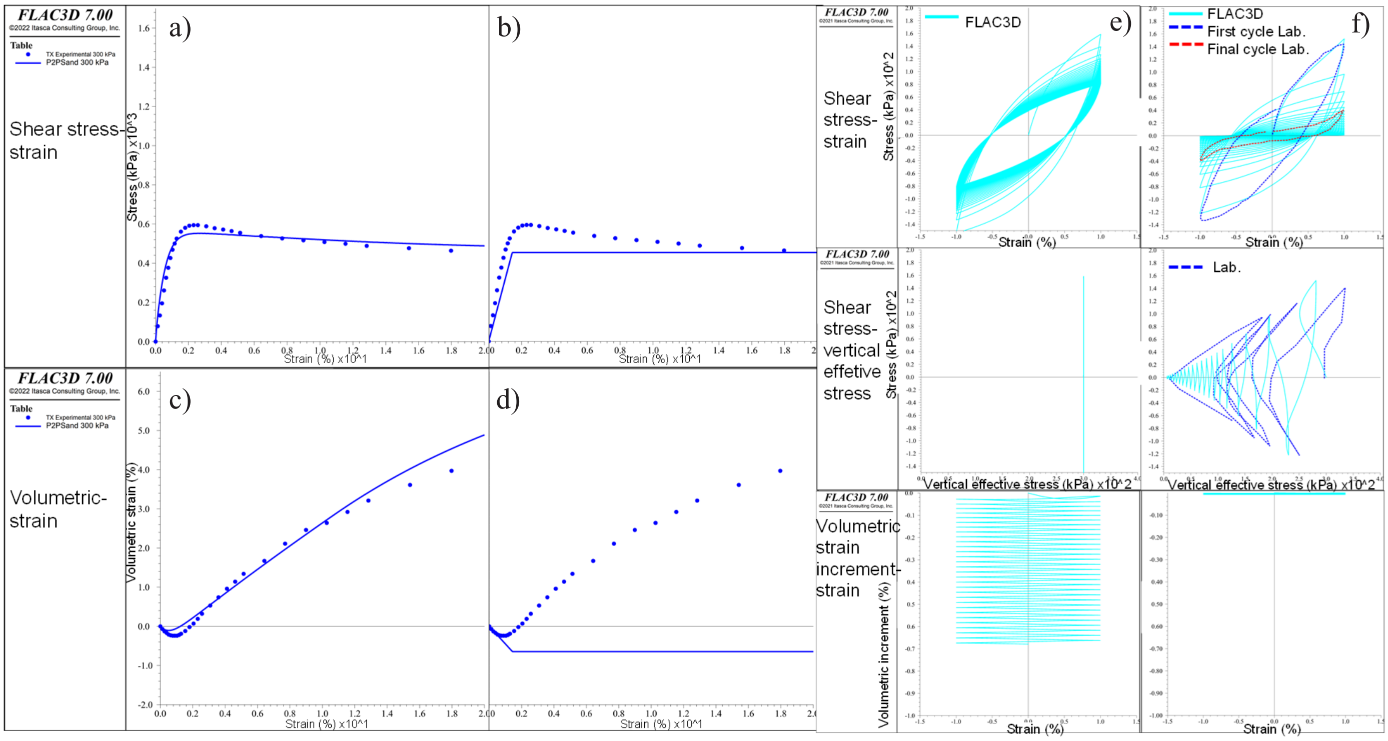

Figure 4: a) Stress-strain P2PSand, b) stress-strain M-C, c) volumetric-strain P2PSand, d) volumetric-strain M-C, e) cyclic drained DSS (300 kPa-1%) and f) cyclic undrained DSS (300 kPa, 1%)

almost perfect agreement is observed, which confirms that the model is capable of reproducing the same behavior depending on two or three parameters. The parameters used as input for the constitutive model P2PSand are shown in Table 3.

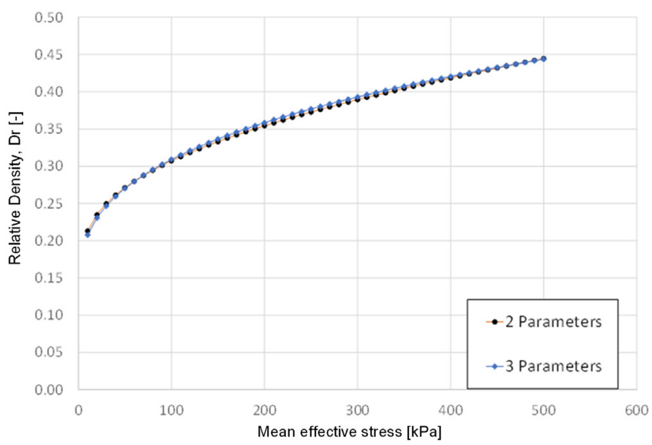

Figure 5: Calibration Critical State Sand Material – Ratio of slopes *Dr* (-) and p_c (kPa) for two and three parameters - P2PSand.

Table 3: Constitutive model P2PSand calibrated properties.

Parameter	Default value sands	Adopted value
K_{cyc}	$3.8-7.2Dr+3Dr^2>0.007$	0.1
k_d	$0.46-0.35Dr$	0.23
φ_{cs}	33°	37°
ν	$0.1+0.015(\varphi_{cs}-25)$	0.27
e_{max}, e_{min}	1.0, 0.6	0.75, 0.40
g_0, C_{Dr}	1240, 0.01	800, 0
Q, R or Dr_{c0}, λr, ξ	10,1 or 10,1,0.07	9.8, 1.6 or 0.12,0.19, 0.33
n^b, n^d	$0.16-\varphi_{cs}/400$, $6n^b = 0.40$	0.0675, 0.27-0.47
z_{max}	$\min(21Dr^{3.85}, 15)$	8
h_o, f, K_{LB}^d	1.7, 0.02, 0.7	1.7, 0.02, 0.7
c	$\frac{3-\sin(\varphi_{cs})}{3+\sin(\varphi_{cs})}$	0.67

Results and discussion

According to the existing evidence regarding the behavior of the main wall during the 2018 earthquake, the observed horizontal and vertical displacements did not exceed 5 cm.

Crest elevation 2947 masl - measured earthquake of M_w = 5.4 - 3D model

Figure 6 depicts a plan view with the residual horizontal and vertical displacement contours that the dam experienced (elevation 2947.5 masl) after the recorded earthquake (M_w = 5.4) for the P2PSand (Figure 6(a)) and Mohr-Coulomb (Figure (b)) constitutive models. Note that, according to the existing evidence about the main dam's performance during the 2018 earthquake, the P2PSand constitutive model fits the observed level of deformations (< 5 cm). According to Figure 7, a comparison between the residual horizontal displacement contours of the two-dimensional and three-dimensional models of the dam (elevation 2947.5 masl) after experiencing a seismic event (M_w = 5.4) for both constitutive models is presented. The results show that the performance of the two-dimensional models is comparable to that of the three-dimensional models.

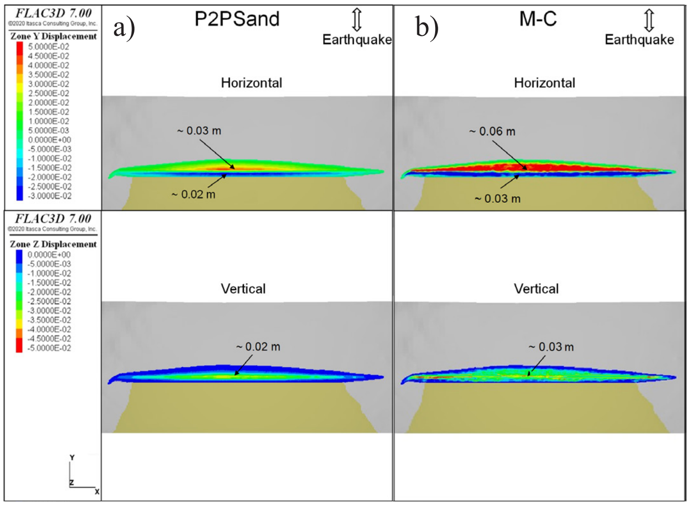

Figure 6: Plan view of horizontal (Y) and vertical (Z) displacement contours (m), a) P2PSand and b) M-C

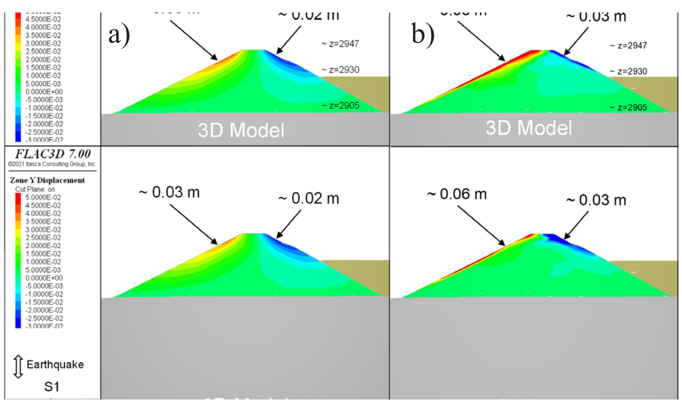

Figure 7: 3D and 2D cross section views of horizontal displacement contours (m), a) P2PSand and b) M-C

Figure 8 shows volumetric and shear strains for drained and undrained condition for P2PSand model.

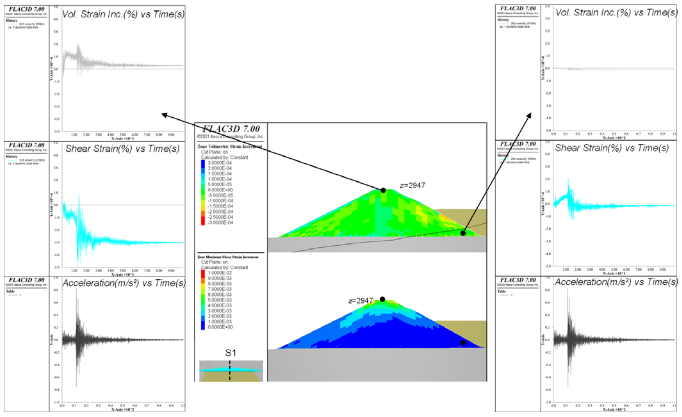

Figure 8: Section view of volumetric and shear strains for P2PSand model. a) Drained and b) undrained condition

Crest elevation 2955 masl - MCE M_w = 8.3 - 3D model

Figure 9 depicts a plan view with the residual horizontal and vertical displacement contours that the dam experienced (elevation 2955 masl) after the MCE (M_w = 8.3) for the P2PSand (Figure 9(a)) and Mohr-Coulomb (Figure 9(b)) constitutive models.

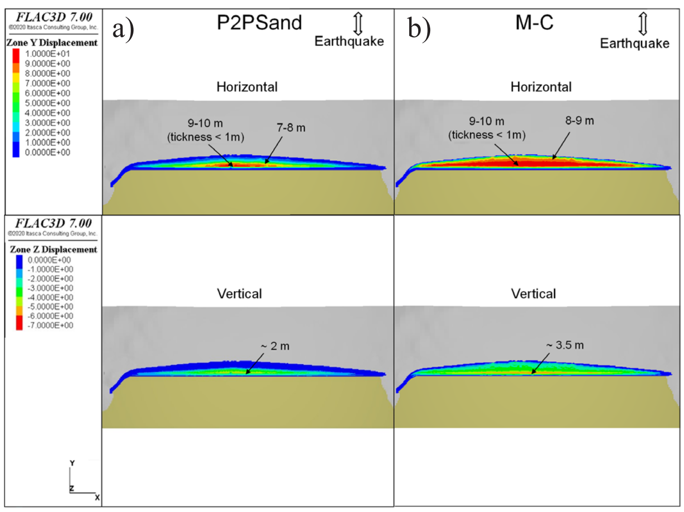

Figure 9: Plan view of residual horizontal and vertical displacement contours (m), a) P2PSand and b) M-C

Crest elevation 2955 masl - sensivity of MCE M_w = 8.3 - 2D model – P2PSand

In addition to the analyzes carried out with the MCE earthquake, a sensitivity analysis was carried out increasing the earthquake by 50 and 100%. Figure 10 shows horizontal acceleration increment, horizontal and vertical displacement contours.

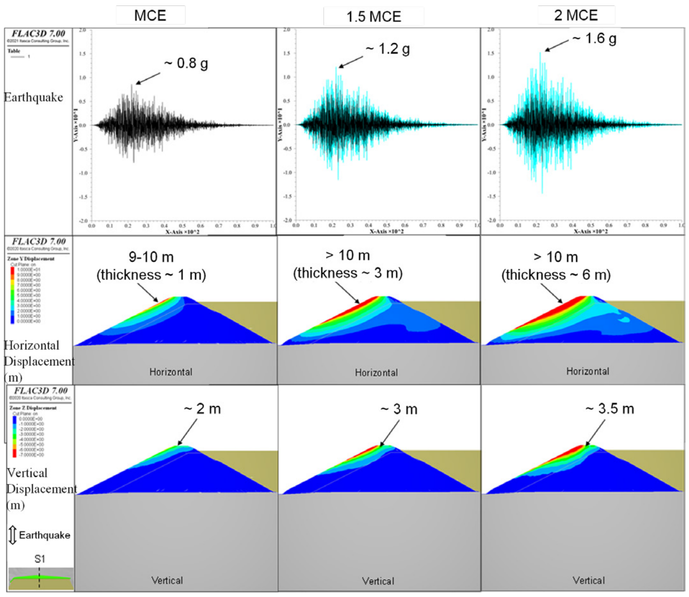

Figure 10: Horizontal acceleration increment, horizontal and vertical displacement contours (m) – P2PSand – Section.

Figure 11 shows the trend of horizontal and vertical displacements in the crest of the wall for the MCE, 1.5 MCE and 2 MCE earthquakes, where an almost linear increase in horizontal displacements can be observed compared to the vertical displacements that attenuate.

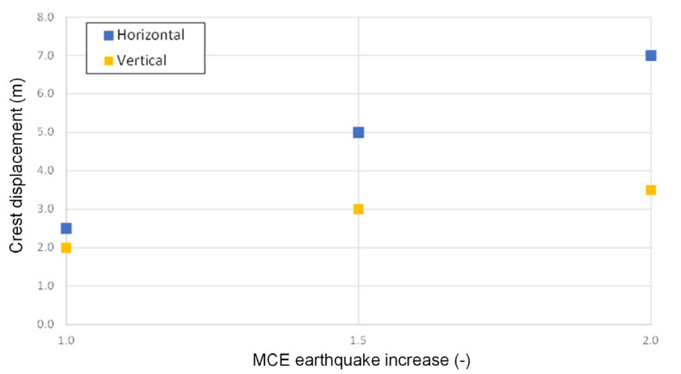

Figure 11: Horizontal and vertical displacements in the crest versus increase in the MCE earthquake.

Conclusions

The developed numerical model follows the standard practice in numerical modeling, incorporating all the available information to simulate the sands' behavior effectively. Based on the results obtained using the advanced P2PSand constitutive model calibration, we conclude that (i) the model reproduces the monotonic response of tailing sand samples observed in laboratory conditions, regarding 100, 300, and 900 kPa of effective confinements, and (ii) the model simulates the drained and undrained tailings sands' cyclic behavior.

The analyzed predictive case (2955 masl elevation) considers the MCE and the MPF, which can be regarded as extreme and unlikely scenarios. Based on the acceptability criteria, the P2PSand constitutive model predicts the dam would suffer severe damage when subjected to such extreme conditions, and the M-C model also predicts the dam's collapsing risk, with longitudinal cracks exceeding a 50 cm width. After analyzing all cases, the outcome for the MCE resulted in an average settlement of 3 m at the crest, which means that, after the seismic event, the freeboard would still be maintained. The results were insensitive to a low Rayleigh damping level (0.2%) for the recorded earthquake and the MCE. Therefore, using more complex constitutive models, such as the P2PSand, is a more reliable and viable option instead of a simpler model, such as the Mohr-Coulomb, which is more commonly used in practice. However, a proper selection and calibration of these constitutive models is crucial. For the advanced models, calibration is achieved by comparing laboratory monotonic and cyclic tests, as well as degradation curves. For the 2018 dam configuration, predictions with the P2PSand constitutive model properly calibrated agree with field observations and recorded instrumental measurements.

P2PSand nomenclature

c	strength ratio
Dr	relative density
Dr_{c0}, λ_c, ξ	3-parameter critical state
e_{max}, e_{min}	maximum and minimum void ratio
f	ratio reverse
g_0, C_{Dr}	elastic parameters
G_r	material parameter
G_{ref}	shape parameter
h_o	plastic shear rate

K^{d}_{LB}	minimum dilatancy ratio
K_{cyc}	factor of cycling
k_{d}	factor of elasticity degradation
n^{b}	coefficient bounding
n^{d}	coefficient dilatancy
p_{atm}	atmospheric pressure
p_{c}	effective pressure
p	mean pressure
Q, R	2-parameter critical state
z_{max}	maximum fabric magnitud
φ_{cs}	critical state friction angle
v	Poisson's ratio

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